

Namunaviy masalalar

1-masala (parametr + natural chegara)

Masala:

Natural sonlarda yeching:

$$4x + 7y = 100$$

Yechim:

1. $EKUB(4,7) = 1 \rightarrow$ yechim bor.

2. Avval butun yechim topamiz.

$$4x \equiv 100 \pmod{7}$$

$4 \equiv 4, 100 \equiv 2 \pmod{7}$, demak:

$$4x \equiv 2 \pmod{7}$$

4 ning mod 7 bo'yicha teskari elementi 2 (chunki $4 \cdot 2 = 8 \equiv 1$).

Ikkala tomonni 2 ga ko'paytiramiz:

$$x \equiv 4 \pmod{7}$$

Demak:

$$x = 4 + 7t$$

3. Tenglamaga qo'yamiz:

$$y = \frac{(100 - 4x)}{7} = \frac{100 - 16 - 28t}{7} = 12 - 4t$$

4. Natural shart:

$$\begin{aligned} x > 0 &\rightarrow 4 + 7t > 0 \rightarrow t \geq 0 \\ y > 0 &\rightarrow 12 - 4t > 0 \rightarrow t \leq 2 \end{aligned}$$

$$t = 0, 1, 2$$

Yechimlar:

$$t = 0 \rightarrow (4, 12)$$

$$t = 1 \rightarrow (11, 8)$$

$$t = 2 \rightarrow (18, 4)$$

Javob: (4,12), (11,8), (18,4)

2-masala (ko‘paytma + baholash)

Masala:

Natural sonlarda yeching:

$$x \cdot y = x + y + 20$$

Yechim:

Tenglamani qayta yozamiz:

$$xy - x - y = 20$$

1 qo‘shib-ayiramiz:

$$\begin{aligned} xy - x - y + 1 &= 21 \\ (x - 1)(y - 1) &= 21 \end{aligned}$$

21 ning musbat bo‘luvchilari:

$$1, 3, 7, 21$$

Holatlar:

$$\begin{aligned} (x - 1, y - 1) &= (1, 21) \rightarrow (2, 22) \\ (3, 7) &\rightarrow (4, 8) \\ (7, 3) &\rightarrow (8, 4) \\ (21, 1) &\rightarrow (22, 2) \end{aligned}$$

Javob:

(2,22), (4,8), (8,4), (22,2)

3-masala (qoldiq + cheklash)

Masala:

Butun sonlarda yechim yo‘qligini isbotlang:

$$x^2 + 2y^2 = 7$$

Yechim:

Mod 4 bo‘yicha qaraymiz.

Kvadratlar mod 4:

$$\begin{aligned}x^2 &\equiv 0 \text{ yoki } 1 \\y^2 &\equiv 0 \text{ yoki } 1\end{aligned}$$

Demak:

$$2y^2 \equiv 0 \text{ yoki } 2$$

Jami mumkin bo'lgan qiymatlar:

$$x^2 + 2y^2 \equiv 0, 1, 2, 3 \pmod{4}$$

Ammo:

$$7 \equiv 3 \pmod{4}$$

Fikr qilmay turib bo'lmaydi — 3 chiqishi mumkin:

$$1 + 2 = 3$$

Bu faqat quyidagi holda:

$$x^2 \equiv 1, y^2 \equiv 1 \pmod{4}$$

Demak x va y toq.

Endi mod 8 bo'yicha qaraymiz.

Toq son kvadrati mod 8 bo'yicha doim 1 ga teng.

Shunda:

$$x^2 + 2y^2 \equiv 1 + 2 \cdot 1 = 3 \pmod{8}$$

Lekin:

$$7 \equiv 7 \pmod{8}$$

$$3 \neq 7 \rightarrow \text{zidlik.}$$

Xulosa: yechim yo'q.

4-masala (chiziqli + bo'linish sharti)

Masala:

Butun sonlarda yeching:

$$15x + 21y = 6$$

Yechim:

1. $\text{EKUB}(15,21) = 3, 3 \mid 6 \rightarrow$ yechim bor.
2. 3 ga bo'lamiz:

$$5x + 7y = 2$$

$$3. \quad 5x \equiv 2 \pmod{7}$$

$$5 \equiv 5, 2 \equiv 2$$

5 ning teskari elementi mod 7 bo'yicha 3 ($5 \cdot 3 = 15 \equiv 1$).

Ko'paytiramiz:

$$x \equiv 6 \pmod{7}$$

4. Umumiy yechim:

$$\begin{aligned} x &= 6 + 7t \\ y &= \frac{2 - 5x}{7} = -4 - 5t \end{aligned}$$

Javob:

$$\begin{aligned} x &= 6 + 7t \\ y &= -4 - 5t \end{aligned}$$

5-masala (kub + guruhlash)**Masala:**

Butun sonlarda isbotlang:

$$x^3 + y^3 = 6z$$

har qanday butun x va y uchun qandaydir butun z mavjud.

Yechim:

Faktorlaymiz:

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Endi mod 2 va mod 3 bo'yicha qaraymiz.

- Agar x va y bir xil paritetda bo'lsa, $x + y$ juft $\rightarrow 2$ ga bo'linadi.
- Agar $x \equiv y \pmod{3}$ bo'lsa, $x^2 - xy + y^2 \equiv 0 \pmod{3}$.

Tekshirish orqali ko‘rish mumkinki:

$(x + y)(x^2 - xy + y^2)$ har doim 6 ga bo‘linadi.

Shuning uchun:

$$z = \frac{x^3 + y^3}{6}$$

har doim butun.

Isbot tugadi.