

MUSTAQIL ISHLASH UCHUN MASALALAR yechimlari

1-masala. Ikki marta og'irlashtirilgan prujina

Shart (eslatma):

$k = 160 \text{ N/m}, m_1 = 0,40 \text{ kg}, A = 0,05 \text{ m}.$

Keyin ustiga yana m_1 qo'yildi $\rightarrow$ umumiy massa $2m_1$, A o'zgarmaydi.

a) Tebranish davrlari T_1 va T_2

Prujinali tebranish davri:

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Dastlab:

$$T^1 = 2\pi \sqrt{\frac{m^1}{k}}$$
$$T_1 = 2\pi \sqrt{\frac{0,40}{160}} = 2\pi \sqrt{0,0025} = 2\pi \cdot 0,05 \approx 0,314 \text{ s}$$

Massani 2 marta oshirsak ($2m_1$):

$$T_2 = 2\pi \sqrt{\frac{2m_1}{k}} = 2\pi \sqrt{2} \cdot \sqrt{\frac{m_1}{k}} = T_1 \sqrt{2}$$
$$T_2 \approx 0,314 \cdot 1,414 \approx 0,444 \text{ s}$$

b) To'liq energiyalar nisbati

To'liq energiya amplitudaga bog'liq:

$$E = \left(\frac{1}{2}\right) k A^2$$

Bu yerda k bir xil, A ham bir xil, demak:

$$E_1 = E_2$$

ya'ni nisbat $E_2 / E_1 = 1$.

Javob:

$T_1 \approx 0,31 \text{ s}, T_2 \approx 0,44 \text{ s}$, energiyalar bir xil ($E_1 = E_2$).

2-masala. Vertikal prujina va osish balandligi

$m = 0,5 \text{ kg}, \Delta l = 0,04 \text{ m}, A = 0,06 \text{ m}, g = 10 \text{ m/s}^2$.

a) Prujina qattiqligi k

Muvozanatda prujina kuchi og'irlikka teng:

$$k \Delta l = m g$$

$$k = m g / \Delta l = 0,5 \cdot 10 / 0,04 = 5 / 0,04 = 125 \text{ N/m}$$

b) Tebranish davri

Prujinali tebranish:

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0,5}{125}}$$

$$\begin{aligned} 0,5 / 125 &= 0,004 \\ \sqrt{0,004} &\approx 0,0633 \end{aligned}$$

$$T \approx 2\pi \cdot 0,0633 \approx 0,40 \text{ s}$$

c) Eng past nuqtadagi cho'zilish

Eng past nuqtada jism muvozanatdan pastga A ga qo'shimcha siljigan:

$$\text{Umumiy cho'zilish} = \Delta l + A = 0,04 + 0,06 = 0,10 \text{ m} = 10 \text{ sm}$$

Javob:

$k = 125 \text{ N/m}, T \approx 0,40 \text{ s}$, eng pastda prujina 10 sm ga cho'zilgan bo'ladi.

3-masala.

$$m = 0,3 \text{ kg}, T_1 = 0,6 \text{ s}.$$

1-qadam: k ni topamiz

$$T_1 = 2\pi \sqrt{\frac{m}{k}}$$

$$k = 4\pi^2 m / T_1^2$$

$k \approx 4 \cdot \pi^2 \cdot 0,3 / 0,6^2 \approx 32,9 \text{ N/m}$ (aniq son shart emas, faqat T_2 uchun kerak)

2-qadam: parallel ulanishdagi $k_{yig'}$

Parallel ulanganida qattqliklar yig'iladi:

$$k_{yig'} = k^1 + k^2 = 2k$$

$$T_2 = 2\pi \sqrt{\frac{m}{k_{yig'}}} = 2\pi \sqrt{\frac{m}{2k}}$$

Bu T_1 bilan bog'liqligi:

$$T_1 = 2\pi \sqrt{\frac{m}{k}}$$

$$T_2 = 2\pi \sqrt{\frac{m}{2k}} = 2\pi \sqrt{\frac{1}{2}} \sqrt{\frac{m}{k}} = T_1 / \sqrt{2}$$

$$T_2 \approx 0,6 / 1,414 \approx 0,42 \text{ s}$$

Javob:

$$T_2 \approx 0,42 \text{ s.}$$

4-masala.

$$A = 0,08 \text{ m}, x = 0,06 \text{ m}, v = 0,8 \text{ m/s}, m = 0,5 \text{ kg.}$$

To'liq energiya doimiy:

$$E = (1/2) k A^2 = (1/2) k x^2 + (1/2) m v^2$$

Bu yerda k noma'lum.

$$(1/2) k (A^2 - x^2) = (1/2) m v^2$$

$$k (A^2 - x^2) = m v^2$$

$$k = m v^2 / (A^2 - x^2)$$

$$A^2 = 0,0064, x^2 = 0,0036 \rightarrow A^2 - x^2 = 0,0028$$

$$k \approx 0,5 \cdot 0,8^2 / 0,0028 = 0,5 \cdot 0,64 / 0,0028 = 0,32 / 0,0028 \\ \approx 114,3 \text{ N/m}$$

a) To'liq energiya

$$E = (1/2) k A^2 \approx 0,5 \cdot 114,3 \cdot 0,0064 \approx 0,3657 \text{ J}$$

$\approx 0,37 \text{ J}$ deb olinadi.

b) k topildi yuqorida: $k \approx 114 \text{ N/m}$

c) Davr

$$T = 2\pi \sqrt{(m / k)} \approx 2\pi \sqrt{(0,5 / 114,3)}$$

$$0,5 / 114,3 \approx 0,00437$$

$$\sqrt{0,00437} \approx 0,0661$$

$$T \approx 2\pi \cdot 0,0661 \approx 0,42 \text{ s}$$

Javob:

$$E \approx 0,37 \text{ J}, k \approx 1,14 \cdot 10^2 \text{ N/m}, T \approx 0,42 \text{ s}.$$

5-masala. Uzunlik o'zgartirilgan mayatnik

$$l_1 = 1,0 \text{ m}, T_2 = 1,5 T_1, g = 9,8 \text{ m/s}^2.$$

Matematik mayatnik davri:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Shu sababli:

$$T \sim \sqrt{l}$$

$$T_2 / T_1 = \sqrt{\frac{l_2}{l_1}}$$

Berilgan: $T_2 / T_1 = 1,5$

$$1,5 = \sqrt{\frac{l_2}{l_1}} \rightarrow l_2 / l_1 = 1,5^2 = 2,25$$

a) $l_2 / l_1 = 2,25$

b) $l_2 = 2,25 \cdot 1,0 = 2,25 \text{ m}$

c) Agar $T_1 = 2,0 \text{ s}$ bo'lsa:

$$T_2 = 1,5 T_1 = 3,0 \text{ s}$$

Javob:

$$l_2/l_1 = 2,25; l_2 = 2,25 \text{ m}; T_2 = 3,0 \text{ s}.$$

6-masala.

$x(t) = 0,10 \cos(4t + \varphi), A = 0,10 \text{ m}, \omega = 4 \text{ rad/s}.$
 $t = 0$ da jism muvozanatdan o'tayapti ($x(0) = 0$) va $v(0) > 0$.

E'tibor: fizik jihatdan $x(0) = 0$ bo'lsa, tezlik moduli ωA bo'lishi kerak, ya'ni $|v(0)| = 4 \cdot 0,1 = 0,4 \text{ m/s}$. Shartdagi $0,2 \text{ m/s}$ soni idealdan farq qiladi, shuning uchun **yo'nalishga mos faza** tanlaymiz (modulni e'tiborsiz qoldiramiz).

a) Boshlang'ich faza φ

$$x(0) = 0,10 \cos(\varphi) = 0 \rightarrow \cos \varphi = 0$$

$$\varphi = \pi/2 \text{ yoki } 3\pi/2 \text{ (yoki } -\pi/2)$$

Tezlik:

$$v(t) = dx/dt = -0,10 \cdot 4 \sin(4t + \varphi) = -0,4 \sin(4t + \varphi)$$

$t = 0$ da:

$$v(0) = -0,4 \sin \varphi$$

$v(0) > 0$ bo'lishi uchun $\sin \varphi < 0$ bo'lishi kerak $\rightarrow \varphi = 3\pi/2$ yoki $-\pi/2$.

Shu bilan:

$$\varphi = -\pi/2 \text{ (qulayroq yoziladi)}$$

b) To'liq tenglama

$$x(t) = 0,10 \cos(4t - \pi/2)$$

Kosinusni sinusga o'tkazsak:

$$\cos(\alpha - \pi/2) = \sin \alpha$$

$$x(t) = 0,10 \sin(4t)$$

c) O'ng chetki holatga birinchi bor yetish vaqti

O'ng chetki holat: $x = +A = 0,10 \text{ m}$.

Bu sinus tenglama bo'yicha $\sin(4t) = 1$ bo'lganda sodir bo'ladi:

$$4t = \pi/2 + 2\pi n$$

Eng kichik musbat t uchun $n = 0$:

$$4t = \pi/2 \rightarrow t = \pi/8 \approx 0,393 \text{ s}$$

Javob:

$\varphi = -\pi/2, x(t) = 0,10 \sin(4t)$, birinchi marta o'ng chetki holatga $t \approx 0,39 \text{ s}$ da yetadi.

7-masala. Fizik mayatnik – tayoqcha

$$l = 0,5 \text{ m}, g = 10 \text{ m/s}^2.$$

a) Massa markazi

Bir jinsli tayoqcha markazi o'rtada:

$$a = l / 2 = 0,25 \text{ m}$$

(bo'lim shartida markaz masofasini shartli tarzda a deb olgandik, bu yerda $a = 0,25 \text{ m}$ bo'ladi.)

b) Inertsia momenti

Ingichka tayoqcha uchiga nisbatan:

$$I = (1/3) m l^2$$

c) Tebranish davri

Fizik mayatnik:

$$T = 2\pi \sqrt{I / (m g a)}$$

O'rniga qo'yamiz:

$$T = 2\pi \sqrt{\frac{\left(\frac{1}{3}\right) m l^2}{m g \left(\frac{l}{2}\right)}}$$

m qisqaradi:

$$T = 2\pi \sqrt{\frac{l^2}{g \frac{l}{2}}} = 2\pi \sqrt{\left(\frac{l^2}{3}\right) \cdot \left(\frac{2}{gl}\right)} = 2\pi \sqrt{\frac{2 l}{3 g}}$$

Endi son qo'yamiz:

$$T = 2\pi \sqrt{\frac{2 \cdot 0,5}{3 \cdot 10}} = 2\pi \sqrt{\frac{1}{30}}$$

$$\frac{1}{30} \approx 0,0333, \sqrt{\frac{1}{30}} \approx 0,1826$$

$$T \approx 2\pi \cdot 0,1826 \approx 1,15 \text{ s}$$

d) Ekvivalent uzunlik

Fizik mayatnik uchun ekvivalent matematik mayatnik uzunligi:

$$l_e = I / (m a)$$

Bu yerda:

$$I = \left(\frac{1}{3}\right) m l^2, a = l/2$$

$$l_e = \left(\left(\frac{1}{3}\right) m l^2\right) / (m l/2) = \left(\frac{1}{3} l^2\right) \cdot (2 / l) = 2l/3$$

$$l_e = 2 \cdot 0,5 / 3 \approx 0,333 \text{ m}$$

Javob:

$T \approx 1,15 \text{ s}$, ekvivalent uzunlik $l_e \approx 0,33 \text{ m}$.

8-masala. So'nuvchi tebranishda amplituda

$A_0 = 10 \text{ sm}$, har 2 davrda amplituda 2 baravar kamayadi.

Bu degani: har $2T$ dan keyin:

$$A \rightarrow A / 2$$

a) 4 davrdan so'ng

4 davr = 2 marta " $2T$ "

Demak:

$$A_4 = A_0 \cdot (1/2)^2 = A_0 / 4 = 10 / 4 = 2,5 \text{ sm}$$

b) 6 davrdan so'ng

6 davr = 3 marta " $2T$ ":

$$A_6 = A_0 \cdot (1/2)^3 = A_0 / 8 = 10 / 8 = 1,25 \text{ sm}$$

c) Energiyaning kamayishi

$$E \sim A^2$$

$$E_6 / E_0 = (A_6 / A_0)^2 = (1/8)^2 = 1/64 \approx 0,016$$

(ya'ni boshlang'ich energiyaning 1,6 % qismi qolgan bo'ladi.)

Javob:

$A_4 = 2,5 \text{ sm}$, $A_6 = 1,25 \text{ sm}$, $E_6/E_0 \approx 1/64$.

9-masala. Bir xil chastotali ikki tebranish

$$x_1(t) = 3 \cos(\omega t)$$

$$x_2(t) = 4 \cos(\omega t)$$

a) Natijaviy tebranish

Bir xil fazali kosinuslar yig'indisi:

$$x(t) = x_1 + x_2 = (3 + 4) \cos(\omega t) = 7 \cos(\omega t)$$

b) Amplituda

$$A = 7$$

c) Maksimal tezlik

$$v(t) = dx/dt = -7 \omega \sin(\omega t)$$

$|v|$ maksimal bo'lganda:

$$v_{max} = 7 \omega$$

Agar $\omega = 5 \text{ rad/s}$ bo'lsa:

$$v_{max} = 7 \cdot 5 = 35 \text{ (birlik/s)}$$

Javob:

$x(t) = 7 \cos(\omega t), A = 7, v_{max} = 35$ ($\omega = 5$ bo'lganda).

10-masala.

$$l = 0,8 \text{ m}, T = 1,8 \text{ s.}$$

Tebranish davri:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Bu yerda g noma'lum.

$$T^2 = 4\pi^2 l / g$$

$$g = 4\pi^2 l / T^2$$

Hisoblaymiz:

$$g \approx 4 \cdot \pi^2 \cdot 0,8 / 1,8^2$$

$$1,8^2 = 3,24$$

$$4\pi^2 \approx 39,48$$

$$g \approx 39,48 \cdot 0,8 / 3,24 \approx 31,58 / 3,24 \approx 9,75 \text{ m/s}^2$$

Xato foizi

Haqiqiy $g_0 = 9,8 \text{ m/s}^2$ deb olaylik.

$$\delta = |g - g_0| / g_0 \cdot 100\% \approx |9,75 - 9,8| / 9,8 \cdot 100\%$$

$$\approx 0,05 / 9,8 \cdot 100\% \approx 0,51 \% \text{ (taxminan yarim foiz)}$$

Javob:

$$g \approx 9,75 \text{ m/s}^2, xato \approx 0,5 \%$$