

NAMUNAVIY MASALALAR

1-masala.

ABC uchburchakda $D \in BC, E \in CA, F \in AB$ shundayki, $BD/DC = 2, CE/EA = 3, AF/FB = 1/5$.

Savol: AD, BE, CF chiziqlar bitta nuqtada kesishadimi?

Yechim: Cheva tekshiramiz $(2) \cdot (3) \cdot \left(\frac{1}{5}\right) = \frac{6}{5} \neq 1$

Xulosa: AD, BE, CF bitta nuqtada kesishmaydi

Metodik signal: Cheva bilan inkor qilish — kuchli usul.

2-masala.

ABC uchburchakda $D \in BC, E \in CA, F \in AB$ (davomlarida bo‘lishi mumkin).

Berilgan: $\frac{BD}{DC} = 4, \frac{CE}{EA} = \frac{1}{2}, \frac{AF}{FB} = \frac{1}{2}$

Isbotlang: D, E, F bitta to‘g‘ri chiziqda yotadi.

Yechim: Menelay: $4 \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) = 1$

Demak: D, E, F kollinear

Metodik signal: Tashqi nuqta bo‘lsa ham qo‘rqma — modul bilan ishlaymiz.

3-masala. (Menelay $\rightarrow$ Cheva)

ABC uchburchakda D, E, F kollinear nuqtalar.

Berilgan: $\frac{BD}{DC} = \frac{AF}{FB}$

Isbotlang: AD, BE, CF bitta nuqtada kesishadi

Yechim: Menelay: $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Shundan: $(CE/EA) = 1 \rightarrow CE = EA$. Demak E — o‘rta nuqta.

Endi Cheva: $\left(\frac{BD}{DC}\right) \cdot (1) \cdot \left(\frac{AF}{FB}\right) = 1$

Keishadi.

Signal: Menelay yordamida Cheva sharti tiklandi.

4-masala. (oraliq uchburchak)

ABC uchburchakda AD, BE, CF lar P da kesishadi. D, E, F mos ravishda BC, CA, AB da.

Isbotlang: $\left(\frac{BP}{PD}\right) \cdot \left(\frac{CP}{PE}\right) \cdot \left(\frac{AP}{PF}\right) = 1$

Yechim (g'oya): Cheva ($\triangle ABC$ da): $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Menelay: $\triangle PBC, \triangle PCA, \triangle PAB$ uchta Menelayni ko'paytirib, soddalashtiramiz.

Natija chiqadi.

Signal: Bu masala 3 ta Menelay talab qiladi — yuqori daraja.

5-masala. (noto'g'ri Cheva tuzog'i)

ABC uchburchakda $D \in BC, E \in CA, F \in AB$. Berilgan $\frac{BD}{DC} = \frac{CE}{EA}$

Savol: AD va BE chiziqlari CF bilan bitta nuqtada kesishadimi?

Yechim: Cheva sharti $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Bu yerda: $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \neq 1$ (umumiy holda) $\frac{AF}{FB}$ noma'lum.

Xulosa: Kesishish kafolatlanmaydi

Signal: 2 ta nisbat yetarli emas — Cheva to'liq bo'lishi kerak.

6-masala. (tashqi Menelay)

ABC uchburchakda D — BC davomida, E — CA ichida, F — AB davomida.

Berilgan: $\frac{BD}{DC} = 2, \frac{CE}{EA} = \frac{1}{3}, \frac{AF}{FB} = \frac{3}{2}$

Isbotlang: D, E, F kollinear

Yechim: Menelay $2 \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{3}{2}\right) = 1$

Kollinear.

7-masala. (ikki uchburchakda ishlash)

ABC uchburchakda P ichki nuqta. AP, BP, CP tomonlarni mos ravishda D, E, F da kesadi.

Isbotlang: $(AD/DB) \cdot (BE/EC) \cdot (CF/FA) = 1$

Yechim: $\triangle ABC$ da bu Cheva emas.

Ammo: AD, BE, CF lar P dan chiqadi

Cheva $\triangle ABC$ emas, $\triangle PBC, \triangle PCA, \triangle PAB$ da ishlatiladi.

Ko'paytirib natija olsak bo'ladi.

Signal: Cheva har doim ABC da yozilmaydi.

8-masala. (inkor bilan Menelay)

ABC uchburchakda D, E, F nuqtalar shundayki, $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) \neq 1$

Isbotlang: D, E, F bitta chiziqda YOTMAYDI

Yechim: Menelay teskari: $\rightarrow$ Agar kollinear bo'lsa, ko'paytma 1 bo'lardi.

Bu 1 emas.

Inkor isbotlandi.

Signal: Bu — Menelayning jonli teskari ishlatilishi.

9-masala. (Cheva + simmetrik nisbatlar)

ABC uchburchakda $D \in BC$ shundayki, $BD = DC, E \in CA, F \in AB$ tanlangan.

Shartni toping: qachon AD, BE, CF kesishadi.

Yechim: Cheva $1 \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Shuning uchun: $\frac{CE}{EA} = \frac{FB}{AF}$

Bu — zarur va yetarli shart.

Signal: Bir nisbat berilsa, qolganlarini bog'lash mumkin.

10-masala. (yakuniy)

ABC uchburchakda $D \in BC, E \in CA, F \in AB$. Berilgan AD, BE, CF kesishadi
Isbotlang: D, E, F + ularning davomlaridagi nuqtalar har qanday Menelay konfiguratsiyasini beradi

Yechim (g'oya): Cheva bajariladi $\left(\frac{BD}{DC}\right) \cdot \left(\frac{CE}{EA}\right) \cdot \left(\frac{AF}{FB}\right) = 1$

Demak: istalgan chiziq orqali, istalgan uchburchakda Menelay ishlaydi.

Bu Cheva $\leftrightarrow$ Menelay ikkiligi.