MUSTAQIL ISHLASH UCHUN MASALALAR YECHIMLARI

1-masala yechimi

$a/(b + c) = a^2/(a(b + c))$ deb yozamiz. Koshi (Engel) ko'rinishi bo'yicha:

$$a^2/(a(b + c)) + b^2/(b(c + a)) + c^2/(c(a + b))$$
$$\geq (a + b + c)^2/(a(b + c) + b(c + a) + c(a + b))$$

Pastki yig'indi:

$$a(b + c) + b(c + a) + c(a + b)$$
$$= 2(ab + bc + ca)$$

Demak:

$$a/(b + c) + b/(c + a) + c/(a + b)$$
$$\geq (a + b + c)^2/(2(ab + bc + ca))$$

Ma'lum tengsizlik:

$$(a + b + c)^2 \geq 3(ab + bc + ca)$$

Shuning uchun:

$$(a + b + c)^2/(2(ab + bc + ca)) \geq 3/2$$

Demak, isbotlandi.

2-masala

$a, b, c > 0$ bo'lsa, isbotlang:
$$(a/b + b/c + c/a)(b/c + c/a + a/b) \geq 9$$

Bu bevosita AM–GM dan:

$$a/b + b/c + c/a \geq 3$$
$$b/c + c/a + a/b \geq 3$$

Demak, ko'paytmasi ≥ 9.

3-masala yechimi

Engel ko'rinishi:

$$a^2/(b + c) + b^2/(c + a) + c^2/(a + b)$$
$$\geq (a + b + c)^2/((b + c) + (c + a) + (a + b))$$

Pastki qism $2(a + b + c)$ ga teng, shuning uchun:

$$\geq (a + b + c)^2/(2(a + b + c)) = (a + b + c)/2$$

Isbotlandi.

4-masala yechimi

$a^2 + b^2 + c^2 = 1$ shart ostida $a + 2b - 2c$ maksimumini topamiz.

Koshi tengsizligi:

$$(a + 2b - 2c)^2 \leq (a^2 + b^2 + c^2) \cdot (1^2 + 2^2 + (-2)^2)$$

$$\leq 1 \cdot (1 + 4 + 4) = 9$$

Demak:

$$a + 2b - 2c \leq 3$$

Maksimum 3.

5-masala yechimi

Koshi (Engel) ko'rinishi:

$$a^2/b + b^2/c + c^2/a \geq (a + b + c)^2/(b + c + a)$$

$$= (a + b + c)^2/(a + b + c) = a + b + c$$

Isbotlandi.