Masalalar yechimlari

1-masala yechimi

$$log_2(3x - 2) = 3$$

AS: $3x - 2 > 0 \rightarrow x > 2/3$.
Yechish: $3x - 2 = 2^3 = 8 \rightarrow 3x = 10 \rightarrow x = 10/3$.
AS ga mos.

Javob: $x = 10/3$

2-masala yechimi

$$log_5(x - 1) = 2$$

AS: $x - 1 > 0 \rightarrow x > 1$.
Yechish: $x - 1 = 5^2 = 25 \rightarrow x = 26$.
AS ga mos.

Javob: $x = 26$

3-masala yechimi

$$log_3(x + 2) + log_3(x - 1) = 1$$

AS: $x + 2 > 0 \, va \, x - 1 > 0 \rightarrow x > 1$.
Yechish: $log_3((x + 2)(x - 1)) = 1$
$$(x + 2)(x - 1) = 3$$
$$x^2 + x - 2 = 3$$
$$x^2 + x - 5 = 0$$

$$x = (-1 \pm \sqrt{21})/2.$$
AS (x > 1) ga mos: $x = (-1 + \sqrt{21})/2$.

Javob: $x = (-1 + \sqrt{21})/2$

4-masala yechimi

$$log_4(x) = log_2(x - 2)$$

AS: $x > 0$ va $x - 2 > 0 \rightarrow x > 2$.
Yechish: $log_4(x) = (1/2) \cdot log_2 x$.
$$(1/2) \cdot log_2 x = log_2(x - 2)$$
$$log_2 x = 2 \cdot log_2(x - 2) = log_2((x - 2)^2)$$

AS da: $x = (x - 2)^2$

$$x = x^2 - 4x + 4$$
$$0 = x^2 - 5x + 4$$
$$(x - 1)(x - 4) = 0$$

AS $(x > 2)$ ga mos: $x = 4$.

Javob: $x = 4$

5-masala yechimi

$$log_2(x + 1) = log_4(2x + 3)$$

AS: $x + 1 > 0$ va $2x + 3 > 0 \rightarrow x > -1/2$.
Yechish: $log_4(2x + 3) = (1/2) \cdot log_2(2x + 3)$.

$$log_2(x + 1) = (1/2) \cdot log_2(2x + 3)$$
$$2 \cdot log_2(x + 1) = log_2(2x + 3)$$
$$log_2((x + 1)^2) = log_2(2x + 3)$$

AS da: $(x + 1)^2 = 2x + 3$

$$x^2 + 2x + 1 = 2x + 3$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

AS $(x > -1/2)$ ga mos: $x = \sqrt{2}$.

Javob: $x = \sqrt{2}$

6-masala yechimi

$$log_3(2x - 1) \geq 2$$

AS: $2x - 1 > 0 \rightarrow x > 1/2$.
Yechish: $3 > 1$, demak:

$$2x - 1 \geq 3^2 = 9$$
$$2x \geq 10$$
$$x \geq 5$$

AS bilan mos.

Javob: $x \geq 5$

7-masala yechimi

$$log_1/_2(x + 4) > 1$$

AS: $x + 4 > 0 \rightarrow x > -4$.
Yechish: $0 < 1/2 < 1$, belgi teskari:
$$x + 4 < (1/2)^1 = 1/2$$
$$x < -7/2$$

AS bilan birlashtiramiz: $-4 < x < -7/2$.

Javob: $-4 < x < -7/2$

8-masala yechimi

$$log_2(x^2 - 4) \leq 3$$

AS: $x^2 - 4 > 0 \rightarrow x < -2$ yoki $x > 2$.
Yechish: $2 > 1$, demak:
$$x^2 - 4 \leq 2^3 = 8$$
$$x^2 \leq 12$$
$$-2\sqrt{3} \leq x \leq 2\sqrt{3}$$

AS bilan kesishma:
$$[-2\sqrt{3}, -2) \cup (2, 2\sqrt{3}]$$

Javob: $[-2\sqrt{3}, -2) \cup (2, 2\sqrt{3}]$

9-masala yechimi

$$log_3(x - 2) = log_9(3x)$$

AS: $x - 2 > 0$ va $3x > 0 \rightarrow x > 2$.
Yechish: $log_9(3x) = (1/2) \cdot log_3(3x)$.
$$log_3(x - 2) = (1/2) \cdot log_3(3x)$$
$$2 \cdot log_3(x - 2) = log_3(3x)$$
$$log_3((x - 2)^2) = log_3(3x)$$

$$AS\,da: (x - 2)^2 = 3x$$
$$x^2 - 4x + 4 = 3x$$
$$x^2 - 7x + 4 = 0$$

$x = (7 \pm \sqrt{33})/2$.
AS (x > 2) ga mos ikkisini tekshiramiz:
$(7 - \sqrt{33})/2 \approx 0{,}63$ (mos emas),
$(7 + \sqrt{33})/2$ mos.

Javob: $x = (7 + \sqrt{33})/2$

10-masala yechimi

$$log_2(x) + log_2(4 - x) = 1$$

AS: $x > 0$ va $4 - x > 0 \rightarrow 0 < x < 4$.
Yechish: $log_2(x(4 - x)) = 1$
$$x(4 - x) = 2$$
$$4x - x^2 = 2$$
$$x^2 - 4x + 2 = 0$$

$$x = (4 \pm \sqrt{16 - 8})/2 = (4 \pm \sqrt{8})/2 = 2 \pm \sqrt{2}.$$
Ikkalasi ham $0 < x < 4$ ga mos.

Javob: $x = 2 - \sqrt{2}, 2 + \sqrt{2}$