

Masalalar yechimlari

1-masala yechimi

Arifmetik progressiya uchun:

$$a_n = a_1 + (n - 1)d$$

Shartlardan foydalanamiz:

$$\begin{aligned} a_2 &= a_1 + d \\ a_5 &= a_1 + 4d \end{aligned}$$

$$a_2 + a_5 = 2a_1 + 5d = 26 \dots (1)$$

$$a_8 = a_1 + 7d = 20 \dots (2)$$

(2) dan $a_1 = 20 - 7d$. Uni (1) ga qo'yamiz:

$$2(20 - 7d) + 5d = 26$$

$$40 - 14d + 5d = 26$$

$$40 - 9d = 26$$

$$9d = 14$$

$$d = 14/9$$

$$a_1 = 20 - 7 \cdot 14/9 = 82/9$$

Javob: $a_1 = 82/9, d = 14/9$

2-masala yechimi

Berilgan: $b_1 + b_4 = 18, b_2 \cdot b_3 = 16$.

Geometrik progressiya: $b_n = b_1 \cdot q^{n-1}$.

$b_4 = b_1 \cdot q^3$, shuning uchun:

$$b_1 + b_4 = b_1 + b_1 \cdot q^3 = b_1(1 + q^3) = 18 \dots (1)$$

$$b_2 \cdot b_3 = (b_1 \cdot q)(b_1 \cdot q^2) = b_1^2 \cdot q^3 = 16 \dots (2)$$

$t = q^3$ deb olamiz. (1) dan $b_1 = 18/(1 + t)$. Buni (2) ga qo'yamiz:

$$(18/(1 + t))^2 \cdot t = 16$$

$$324t/(1 + t)^2 = 16$$

$$81t/(1 + t)^2 = 4$$

$$81t = 4(1 + 2t + t^2)$$

$$81t = 4 + 8t + 4t^2$$

$$4t^2 - 73t + 4 = 0$$

$$t = (73 \pm 9\sqrt{65})/8, \text{ ya'ni:}$$

$$q^3 = (73 \pm 9\sqrt{65})/8$$

$$q = \sqrt[3]{((73 \pm 9\sqrt{65})/8)}$$

$b_1 = 18/(1 + t)$ dan chiroyli ko‘rinishda:

- Agar $q^3 = (73 - 9\sqrt{65})/8$ bo‘lsa, $b_1 = 9 + \sqrt{65}$
- Agar $q^3 = (73 + 9\sqrt{65})/8$ bo‘lsa, $b_1 = 9 - \sqrt{65}$

Javob (2 ta yechim):

1. $b_1 = 9 + \sqrt{65}, q = \sqrt[3]{((73 - 9\sqrt{65})/8)}$
2. $b_1 = 9 - \sqrt{65}, q = \sqrt[3]{((73 + 9\sqrt{65})/8)}$

3-masala yechimi

Berilgan: arifmetik progressiyada manfiy hadlar soni 6 ta, $a_1 = 5, d < 0$.
Oxirgi manfiy had so‘ralgan.

Lekin $d < 0$ va $a_1 = 5 > 0$ bo‘lsa, progressiya kamayib boradi va biror joydan keyin manfiy bo‘lib qolsa, **undan keyingi barcha hadlar ham manfiy bo‘ladi**.
 Ya’ni manfiy hadlar soni **cheksiz** bo‘ladi.

Shuning uchun “manfiy hadlar soni 6 ta” sharti bilan “ $d < 0, a_1 = 5$ ” sharti **o‘zaro zid**; bunday progressiyada “oxirgi manfiy had” mavjud emas.

Xulosa: berilgan shartlarda masala yechimga ega emas (oxirgi manfiy had yo‘q).

4-masala yechimi

Berilgan: $b_1 > 0, q > 1, b_1 + b_2 + b_3 = 21, b_3 - b_1 = 12$.
 $b_2 = b_1q, b_3 = b_1q^2$.

Shartlar:

$$\begin{aligned} b_1(1 + q + q^2) &= 21 \dots (1) \\ b_1(q^2 - 1) &= 12 \dots (2) \end{aligned}$$

(2) dan $b_1 = 12/(q^2 - 1)$. (1) ga qo‘yamiz:

$$\begin{aligned} 12(1 + q + q^2)/(q^2 - 1) &= 21 \\ 4(1 + q + q^2)/(q^2 - 1) &= 7 \\ 4 + 4q + 4q^2 &= 7q^2 - 7 \\ 3q^2 - 4q - 11 &= 0 \end{aligned}$$

$$D = 16 + 132 = 148 = 4 \cdot 37 \Rightarrow \sqrt{D} = 2\sqrt{37}$$

$$q = (4 + 2\sqrt{37})/6 = (2 + \sqrt{37})/3 \text{ (} q > 1 \text{ shundan)}$$

$b_1 = 12/(q^2 - 1)$ hisoblab chiqilsa:

$$\mathbf{b_1 = 8 - \sqrt{37}}$$

Endi:

$$b_2 = b_1q = (8 - \sqrt{37}) \cdot (2 + \sqrt{37})/3 = -7 + 2\sqrt{37}$$

$$b_3 = b_1q^2 = 20 - \sqrt{37}$$

Javob (progressiya):

$$b_1 = 8 - \sqrt{37}, b_2 = -7 + 2\sqrt{37}, b_3 = 20 - \sqrt{37}, q = (2 + \sqrt{37})/3.$$

5-masala yechimi

Cheksiz geometrik progressiya:

$$b_1 = 6, b_2 = 3 \rightarrow q = 3/6 = 1/2$$

$|q| < 1$, demak yig'indi mavjud.

$$S = b_1/(1 - q)$$

$$S = 6/(1 - 1/2) = 12$$

Javob: 12