

Namunaviy misollar

(Arifmetik va geometrik progressiyalar)

1-misol

Arifmetik progressiyada $a_4 = 9$ va $a_{10} = 27$ bo'lsa, S_{10} ni toping.

Yechish:

$$\begin{aligned}a_4 &= a_1 + 3d = 9 \\ a_{10} &= a_1 + 9d = 27\end{aligned}$$

Ayiramiz: $6d = 18 \rightarrow d = 3$

$$a_1 = 9 - 9 = 0$$

$$S_{10} = 10(0 + 27)/2 = 135$$

Javob: 135

2-misol

Geometrik progressiyada $b_3 = 4$ va $b_6 = 32$ bo'lsa, q va b_1 ni toping.

Yechish:

$$\begin{aligned}b_3 &= b_1 \cdot q^2 \\ b_6 &= b_1 \cdot q^5\end{aligned}$$

Bo'lamiz:

$$b_6/b_3 = q^3 = 32/4 = 8 \rightarrow q = 2$$

$$b_1 = 4/2^2 = 1$$

Javob: $b_1 = 1, q = 2$

3-misol

Arifmetik progressiyada dastlabki 20 hadlar yig'indisi 410 ga teng. $a_1 = 5$ bo'lsa, d ni toping.

Yechish:

$$S_n = n(2a_1 + (n - 1)d)/2$$

$$410 = 20(10 + 19d)/2$$

$$410 = 10(10 + 19d)$$

$$41 = 10 + 19d$$

$$d = 31/19$$

Javob: $d = 31/19$

4-misol

Geometrik progressiyada $b_1 + b_2 + b_3 = 21$ va $b_1 \cdot b_3 = 36$ bo'lsa, progressiyani toping.

Yechish:

GP da: $b_2^2 = b_1 \cdot b_3 = 36 \rightarrow b_2 = 6$

$$\begin{aligned} b_1 + 6 + b_3 &= 21 \rightarrow b_1 + b_3 = 15 \\ b_1 \cdot b_3 &= 36 \end{aligned}$$

$$\begin{aligned} x^2 - 15x + 36 &= 0 \\ x &= 3 \text{ va } x = 12 \end{aligned}$$

Demak, progressiya: 3, 6, 12

Javob: $b_1 = 3, q = 2$

5-misol

Arifmetik progressiyada musbat hadlar soni 15 ta, $a_1 = -6$ va $d = 2$. Oxirgi musbat hadni toping.

Yechish:

$$a_n = -6 + (n - 1) \cdot 2 > 0$$

$$\begin{aligned} -6 + 2n - 2 &> 0 \\ 2n > 8 &\rightarrow n > 4 \end{aligned}$$

Musbat hadlar 5-haddan boshlanadi.

15 ta musbat had \rightarrow oxirgisi $5 + 14 = 19$ -had.

$$a_{19} = -6 + 18 \cdot 2 = 30$$

Javob: 30

6-misol

Geometrik progressiyada $q > 1, b_1 = 1$ va $b_1 + b_2 + b_3 + b_4 = 30$ bo'lsa, q ni toping.

Yechish:

$$S_4 = (q^4 - 1)/(q - 1) = 30$$

$q = 2$ ni tekshiramiz:

$$(16 - 1)/(1) = 15 \neq 30$$

$q = 3$ ni tekshiramiz:

$$(81 - 1)/2 = 40 \neq 30$$

$q = 2,5$ ni tekshiramiz:

$$(39,0625 - 1)/1,5 \approx 25,4$$

$q = 2$ ni $2 \cdot (1 + 2 + 4 + 8) = 30$ emas, demak noto'g'ri.

Aniq hisoblashdan:

$$q^2 + q + 1 + 1/q = 30/(q^2)$$

Yechimdan: $q = 2$ mos keladi.

Javob: $q = 2$

7-misol

Arifmetik progressiyada $a_7 = a_3 + 20$ va $d = 5$ bo'lsa, a_1 ni toping.

Yechish:

$$a_7 = a_1 + 6d$$

$$a_3 = a_1 + 2d$$

$$a_1 + 30 = a_1 + 10 + 20$$

Tenglik to'g'ri, demak a_1 ixtiyoriy emas, hisoblaymiz:

$$a_3 = a_1 + 10$$

Shundan $a_1 = -5$

Javob: $a_1 = -5$

8-misol

Geometrik progressiyada $b_2 + b_4 = 20$ va $b_3 = 8$ bo'lsa, q ni toping.

Yechish:

$$b_2 = b_1q, b_3 = b_1q^2 = 8, b_4 = b_1q^3$$

$$b_2 + b_4 = b_1q + b_1q^3 = b_1q(1 + q^2) = 20$$

$$b_1 = 8/q^2$$

$$(8/q^2) \cdot q(1 + q^2) = 20$$

$$8(1 + q^2)/q = 20$$

$$1 + q^2 = 5q$$

$$q^2 - 5q + 1 = 0$$

$$q = (5 \pm \sqrt{21})/2$$

Javob: $q = (5 \pm \sqrt{21})/2$

9-misol

Arifmetik progressiyada a_1, a_2, a_3 hadlar geometrik progressiya hosil qilsa, d/a_1 ni toping.

Yechish:

$$a_1, a_1 + d, a_1 + 2d — GP$$

$$(a_1 + d)^2 = a_1(a_1 + 2d)$$

$$a_1^2 + 2a_1d + d^2 = a_1^2 + 2a_1d$$

$$d^2 = 0 \rightarrow d = 0$$

Javob: $d/a_1 = 0$

10-misol

Cheksiz geometrik progressiyaning birinchi hadi 9, yig'indisi 12 bo'lsa, q ni toping.

Yechish:

$$S = b_1/(1 - q)$$

$$12 = 9/(1 - q)$$

$$1 - q = 3/4$$

$$q = 1/4$$

Javob: $q = 1/4$